MELTING OF A SEMIINFINITE STEEL BODY IN AN IRON–CARBON ALLOY

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UDC 536.421.1

Melting of a semiinfinite steel body in an iron–carbon alloy is considered. A solution is derived for equations of heat and mass exchange which are obtained by means of the integral Laplace transform.

The process of melting of steel scrap which takes place in open hearth furnaces and in converters is little studied at the present time. The peculiarity of this process is that owing to the carburization the solid metal becomes liquid at a temperature which is much lower than its melting temperature. In [1, 2] the transition process of the steel scrap into a liquid state is considered purely as a diffusion process. However, the authors of [4] note that the actual rate of this process applied to molybdenum is 3-4 orders higher than according to the calculations based on the diffusion theory. In [3, 5, and 6] the process of transition of the steel into a liquid state in markedly carburizing melts is considered as melting with simultaneous progressing of the carbon diffusion process from the melt to the surface of the metal, which results in the reduction of the melting temperature of its surface. Hence the linear rate of melting depends on both the intensity of the diffusion of the carbon from the melt to the surface of the scrap and hence also on the heat exchange between the liquid and solid phases.

The melting of the semiinfinite steel body in the liquid iron–carbon alloy is considered below. Melting and heating of the body take place as a result of convective heat exchange between the molten material and the surface of the body. The process of carburization of the body proceeds as a parallel process. It is assumed that at each instant of time the temperature of the external layer is determined by the liquidus line of the iron–carbon composition diagram. The indicated process is described by the following system of equations:

$$\frac{\partial T}{\partial \tau} = a\frac{\partial^2 T}{\partial x^2}, \quad \xi(\tau) \leqslant x < \infty, \tag{1}$$

$$\frac{\partial C}{\partial \tau} = D\frac{\partial^2 C}{\partial x^2}, \quad \xi(\tau) \leqslant x < \infty, \tag{2}$$

with the initial and boundary conditions

$$\alpha(T_m - T_s) = \gamma q\frac{d\xi(\tau)}{d\tau} - \lambda\frac{\partial T}{\partial x}, \quad x = \xi(\tau), \tag{3}$$

$$\beta(C_m - C_s) = \frac{d\xi(\tau)}{d\tau}(C_s - C_0) - D\frac{\partial C}{\partial x}, \quad x = \xi(\tau), \tag{4}$$

$$\frac{\partial T}{\partial x} = 0, \quad x \to \infty, \tag{5}$$

$$\frac{\partial C}{\partial x} = 0, \quad x \to \infty, \tag{6}$$

$$T(x;\ 0) = 0, \tag{7}$$

$$C(x;\ 0) = C_0. \tag{8}$$

The values of T_m and C_m are considered as constant with time.

Institute of Metallurgical Thermal Technology, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 17, No. 5, pp. 931-935, November, 1969. Original article submitted January 2, 1969.

The equation of the liquidus line of the iron–carbon diagram is written in the form

$$T_s = T^0_{\mathrm{M}} - m\,(C_s - C_0), \tag{9}$$

Where T^0_{M} is the temperature of the liquidus at the concentration C_0; m is the factor determining the increase of the temperature of the system on reducing the concentration by 1%.

The established problem (1) to (8) is related to the class of problems which are called the Stefan problem in the specialist literature. For such problems the temperature field is continuous, but the temperature gradient undergoes a discontinuity which makes this problem a nonlinear one with a weak discontinuity on the boundary.

We will divide the time of the melting into such small intervals $\Delta\tau_i = (\tau_{i+1} - \tau_i)$ that the rate of the progress of the melting front at that interval of time can be taken as constant: $d\xi/d\tau = \dot{\xi} = \mathrm{const}$. Calculations carried out by I. V. Belov [6] showed that as a result of the small magnitude of the diffusion coefficient D of carbon in the solid phase, and the sufficiently high rates of travel of the boundary, the distribution of the concentration in the body is very little dependent on the rate of movement of the boundary and it will remain practically the same as for the initial interval of time $\Delta\tau_1$.

In connection with this, in order to find the distribution of concentration it is possible to take advantage of the known solution of É. M. Gol'dfarb [7], which is obtained for the case of melting of a semiinfinite body at constant melting rate:

$$\frac{C(x;\ \tau) - C_0}{C_s - C_0} = \frac{1}{2}\left\{\exp\left[-\left(\frac{\dot{\xi}_1 x}{D} - \frac{\dot{\xi}_1^2\tau}{D}\right)\right]\mathrm{erfc}\left(\frac{x}{2\sqrt{D\tau}} - \sqrt{\frac{\dot{\xi}_1^2\tau}{D}}\right) + \mathrm{erfc}\left(\frac{x}{2\sqrt{D\tau}}\right)\right\}. \tag{10}$$

Differentiating (10) with respect to x, we find the magnitude D ∂C/∂X on the traveling boundary

$$D\,\frac{\partial C}{\partial x} = D\,\frac{C_s - C_0}{2}\left\{-\frac{\dot{\xi}_1}{D}\,\mathrm{erfc}\left(-\frac{1}{2}\sqrt{\frac{\dot{\xi}_1^2\tau}{D}}\right) - \frac{4}{\sqrt{\pi}}\exp\left(-\frac{\dot{\xi}_1^2\tau}{4D}\right)\right\}. \tag{11}$$

On substituting (11) into Eq. (4) we will find the value of the concentration C_s of the carbon in the surface layer

$$C_s = \frac{C_0\left(\frac{D}{2}\Psi - \dot{\xi}_1\right) - \beta C_{\mathrm{m}}}{\left(\frac{D}{2}\Psi - \dot{\xi}_1\right) - \beta}, \tag{12}$$

where

$$\Psi = -\frac{\dot{\xi}_1}{D}\,\mathrm{erfc}\left(-\frac{1}{2}\sqrt{\frac{\dot{\xi}_1^2\tau}{D}}\right) - \frac{4}{\sqrt{\pi}}\exp\left(-\frac{\dot{\xi}_1^2\tau}{4D}\right).$$

In order to establish the temperature field in the interval $\Delta\tau_i$ we will introduce a moving system of coordinates relative to which the front of melting is stationary:

$$z = x - \dot{\xi}_i\tau;\quad \tau = \tau.$$

The equation of thermal conductivity (1) in such a system of calculations has the form of a "convection" equation:

$$a\,\frac{\partial^2 T}{\partial z^2} + \dot{\xi}_i\,\frac{\partial T}{\partial z} = \frac{\partial T}{\partial \tau} \tag{13}$$

with conditions

$$T(0;\ \tau_i) = T_s;\quad T(z;\ \tau_{i-1}) = T_{i-1}(z);\quad T(\infty;\ \tau_i) = 0. \tag{14}$$

We note that in the instant of time $\tau = 0$ the temperature on the surface of the body rises with a jump to melting temperature T_s.

By using the Laplace integral transform

$$F(p) = \int_0^\infty f(\tau)\exp(-p\tau)\,d\tau, \tag{15}$$

we obtain in the representation

$$aT''_L - \dot{\xi}_i T'_L = p\left(T_L - \frac{T_{i-1}(z)}{p}\right) \tag{16}$$

with conditions

$$T_L(0;\ p) = \frac{T_s}{p};\quad T_L(\infty;\ p) = 0. \tag{17}$$

The solution of (16) is of the form

$$\begin{aligned}
T_L(z;\ p) = {} & \frac{T_s}{p}\exp(r_2 z) - \exp(r_2 z)\int\limits_0^{\infty}\frac{T_{i-1}}{a(r_1 - r_2)}\exp(-r_1\varepsilon)\,d\varepsilon \\
& + \exp(r_1 z)\int\limits_0^{\infty}\frac{T_{i-1}}{a(r_1 - r_2)}\exp(-r_1\varepsilon)\,d\varepsilon - \exp(r_1 z)\int\limits_0^{z}\frac{T_{i-1}}{a(r_1 - r_2)} \\
& \times \exp(-r_1\varepsilon)\,d\varepsilon + \exp(r_2 z)\int\limits_0^{z}\frac{T_{i-1}}{a(r_1 - r_2)}\exp(-r_2\varepsilon)\,d\varepsilon,
\end{aligned} \tag{18}$$

where $r_{1,2} = -\frac{\dot{\xi}_i}{2a} \pm \sqrt{\frac{\dot{\xi}_i^2}{4a^2} + \frac{p}{a}}$ is the root of the characteristic equation.

On transition from the representations to originals by means of the tables [8], we find in the moving system of the calculation:

$$\begin{aligned}
T(z;\ \tau_i) = {} & T_s\,\frac{1}{2}\left[\exp\left(\frac{\dot{\xi}_i z}{a}\right)\operatorname{erfc}\left(\frac{z}{2\sqrt{a\tau_i}} - \sqrt{\frac{\dot{\xi}_i^2\tau_i}{4a}}\right)\right. \\
& \left. + \operatorname{erfc}\left(\frac{z}{2\sqrt{a\tau_i}} + \sqrt{\frac{\dot{\xi}_i^2\tau_i}{4a}}\right)\right] + \frac{1}{2\sqrt{\pi a\tau_i}}\exp\left(-\frac{\dot{\xi}_i\tau_i}{4a}\right) \\
& \times\left[\int\limits_0^{z} T_{i-1}\exp\left[\frac{\dot{\xi}_i}{2a}(\varepsilon - z)\right]\left\{\exp\left[-\frac{(z-\varepsilon)^2}{4a\tau_i}\right] - \exp\left[\frac{(\varepsilon+z)^2}{4a\tau_i}\right]\right\}d\varepsilon\right. \\
& \left. + \int\limits_z^{\infty} T_{i-1}\exp\left[\frac{\dot{\xi}_i}{2a}(\varepsilon - z)\right]\left\{\exp\left[-\frac{(\varepsilon-z)^2}{4a\tau_i}\right] - \exp\left[-\frac{(\varepsilon+z)^2}{4a\tau_i}\right]\right\}d\varepsilon\right],
\end{aligned} \tag{19}$$

and in the stationary system of calculation

$$\begin{aligned}
T(x;\ \tau_i) = {} & T_s\frac{1}{2}\left\{\exp\left[-\left(\frac{\dot{\xi}_i x}{a} - \frac{\dot{\xi}_i^2\tau_i}{a}\right)\right]\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau_i}} - \sqrt{\frac{\dot{\xi}_i^2\tau_i}{a}}\right)\right. \\
& \left. + \operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau_i}}\right)\right\} + \frac{1}{2\sqrt{\pi a\tau_i}}\exp\left(-\frac{\dot{\xi}_i\tau_i}{4a}\right) \\
& \times\left[\int\limits_{\dot{\xi}_i\tau_i}^{x-\dot{\xi}_i\tau_i} T_{i-1}\exp\left[\frac{\dot{\xi}_i}{2a}(\varepsilon - x + \dot{\xi}_i\tau_i)\right]\left\{\exp\left[-\frac{(x - \dot{\xi}_i\tau_i - \varepsilon)^2}{4a\tau_i}\right]\right.\right. \\
& \left. - \exp\left[-\frac{(\varepsilon + x - \dot{\xi}_i\tau_i)^2}{4a\tau_i}\right]\right\}d\varepsilon + \int\limits_{x-\dot{\xi}_i\tau_i}^{\infty} T_{i-1}\exp\left[\frac{\dot{\xi}_i}{2a}(\varepsilon - x + \dot{\xi}_i\tau_i)\right] \\
& \left. \times\left\{\exp\left[-\frac{(\varepsilon - x + \dot{\xi}_i\tau_i)^2}{4a\tau_i}\right] - \exp\left[-\frac{(\varepsilon + x - \dot{\xi}_i\tau_i)^2}{4a\tau_i}\right]\right\}d\varepsilon\right].
\end{aligned} \tag{20}$$

The values of the temperatures of melting will be found, by substituting C_S into (9):

$$T_s = T^0_{\mathrm{M}} - m\beta\,\frac{(C_{\mathrm{m}} - C_0)}{\frac{D}{2}\,\Psi - \dot{\xi}_1 - \beta}\,. \tag{21}$$

The value of $\dot{\xi}$ can be found by using expressions (3), (20), and (21)

$$\dot{\xi}_i = \frac{\alpha}{\gamma q}\left[T_{\mathrm{m}} - T^0_{\mathrm{M}} + m\beta\,\frac{C_{\mathrm{m}} - C_0}{\frac{D}{2}\,\Psi - \dot{\xi}_1 - \beta}\right] + \frac{\lambda}{\gamma q}\,\frac{\partial T_i}{\partial x}\,. \tag{22}$$

The average value of the rate of melting is given by:

$$\dot{\xi}_{\mathrm{av}} = \frac{\sum\limits_{i=1}^{i=n} \left\{ \frac{\alpha}{\gamma q} \left[T_{\mathrm{m}} - T_{\mathrm{M}}^{0} + m\beta \frac{C_{\mathrm{m}} - C_0}{\frac{D}{2}\Psi - \dot{\xi}_1 - \beta} \right] + \frac{\lambda}{\gamma q} \frac{\partial T_i}{\partial x} \right\} \Delta\tau_i}{\sum\limits_{i=1}^{i=n} \Delta\tau_i} . \tag{23}$$

The solution obtained characterizes the dynamics of the progress of the melting process for nonisothermal conditions. Both the phenomenon of heat transfer, and the diffusion from the melt to the surface of the melting body are studied. The formulae obtained make it possible to determine the rate of melting successively for separate intervals of time, starting with the first. The average rate of melting after time τ is determined by the formula (23).

NOTATION

a	is the thermal diffusivity;
λ	is the thermal conductivity;
D	is the diffusivity;
α, β	are the heat and mass transfer coefficients respectively;
T_s	is the melting temperature of surface layer;
$\xi(\tau)$	is the movable boundary coordinate;
T_m, C_m	are the temperature and concentration in the melt.

LITERATURE CITED

1. A. N. Morozov, O. Ya. Vainshtein et al., Transactions of the All-Union Scientific Technological Conference on the Application of Radioactive and Stable Isotopes and Irradiations in the National Economy and Science. Metallurgy and Metal Science [in Russian], Akad. Nauk SSSR (1958).
2. L. A. Shvartsman, A. I. Osipov et al., in: Problems of Metal Science and Physics of Metals, No. 6 [in Russian], Central Scientific Research Institute of Ferrous Metallurgy, Institute of Metal Science and Physics of Metals (1959).
3. V. S. Zhernovskii, E. A. Kapustin, et al., in: Production of Steel, Central Scientific Research Institute of Ferrous Metallurgy, No. 4 (1966).
4. L. N. Krupman and V. I. Yavoiskii, Reports from Higher Educational Establishments, Chernaya Metallurgiya, No. 7 (1965).
5. E. A. Kapustin, Reports from Higher Educational Establishments, Chernaya Metallurgiya, No. 7 (1965).
6. I. V. Belov, Izv. Akad. Nauk SSSR, Metally, No. 6 (1966).
7. É. M. Gol'dfarb, Thermal Technology of Metallurgical Processes [in Russian], Metallurgiya, Moscow (1967).
8. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).